

QUARK FRAGMENTATION FUNCTIONS IN LOW-ENERGY CHIRAL THEORY*

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(MIT-CTP-2259. Submitted to: *Phys. Rev. Lett.* November 1993)

We examine the physics content of fragmentation functions for inclusive hadron production in a quark jet and argue that it can be calculated in low energy effective theories. As an example, we present a calculation of u -quark fragmentation to π^+ and π^- mesons in the lowest order in the chiral quark model. The comparison between our result and experimental data is encouraging.

Despite lack of a rigorous proof, many believe that the color charges in Quantum Chromodynamics (QCD) are permanently confined. The building blocks of QCD, quarks and gluons, cannot emerge as asymptotic states of the theory and thus are not directly detectable in an experiment. Rather, traces of energetic quarks and gluons in a hard collision manifest in jets of hadrons with highly correlated momenta. Since their discovery in 1975, jets have become bread-and-butter physics in high-energy colliders.

Parton fragmentation refers to the process of converting high-energy, colored quarks and gluons out of a hard scattering into hadron jets observed in detectors. Undoubtedly, this process involves QCD physics at many different scales and is rather complicated. However, important developments in perturbative QCD occurred in the beginning of 80's, coupling with rich experimental data taken from high-energy colliders, have taught us a great deal about what is going on in fragmentation process [1]. In a modern view, parton fragmentation involves three key concepts: separation of short and long distance physics (factorization theorem or assumption), perturbative evolution of partons from high to low virtualities (parton shower), and non-perturbative fragmentation of partons with virtuality of order of 1 GeV to hadrons (hadronization). While the first two subjects can be treated systematically in perturbation theory, the last one is intrinsically non-perturbative and is difficult to study directly in QCD. In the past, phenomenological models, such as Feynman-Field model [2] or Lund string model [3], have been used to describe hadronization in Monte Carlo simulations. Except for heavy quarks [4], little progress has been made on understanding fragmentation physics from the fundamental theory.

In this Letter we attempt to study hadronization from a low energy effective theory, focusing on calculating fragmentation functions for inclusive hadron production. In order for the reader to understand the context of our calculation and to disperse possible doubts over its relevance, we begin with inclusive hadron production in e^+e^- annihilation, for which a factorization theorem can be proved rigorously in perturbative QCD [5,6]. The theorem asserts that in the leading order in hard momentum the inclusive hadron is produced by fragmentation of a *single* quark without influence of others (independent jet fragmentation). Consequently, the fragmentation functions, which describe hadron distributions in the jet, can be expressed in terms of the matrix elements of the quark field operator alone. If similar factorization theorems can be proved for other processes, the same functions appear in the relevant hadron-production cross sections.

Like parton distribution functions, the parton fragmentation functions are scale dependent, and the scale evolution is governed by renormalization group equations [5]. At low-energy scales, the fragmentation functions contain no large momenta and shall be calculable in low-energy models. To illustrate this, we consider pion production in a quark jet using the chiral quark model of Manohar and Georgi [7]. The tree level result for π^+ and π^- productions in a u -quark jet shows an impressive similarity with the EMC data when evolved to appropriate energy scales. The higher-order corrections can be taken into account systematically in a chiral expansion.

To begin our discussion, we consider the hadron tensor for inclusive hadron production in e^+e^- annihilation,

$$\hat{W}_{\mu\nu} = \frac{1}{4\pi} \sum_X \int d^4\xi e^{iq\cdot\xi} \langle 0 | J_\mu(\xi) | H(P) X \rangle \langle H(P) X | J_\nu(0) | 0 \rangle \quad (1)$$

*This work is supported in part by funds provided by the U.S. Department of Energy (D.O.E.) under contract #DE-AC02-76ER03069 (MIT) and the National Science Foundation (Drexel).

where q is the moment of a time-like virtual photon, P is the moment of the observed hadron H and X represents other unobserved hadrons and is summed over. In the following discussion, we choose a special coordinate system defined by two light-cone vectors $p = \mathcal{P}(1, 0, 0, 1)$ and $n = 1/(2\mathcal{P})(1, 0, 0, -1)$ with $p \cdot n = 1$, in which the hadron and photon momenta are collinear: $P = p + nM^2/2$, $q = p/z + \nu n$. In the deep-inelastic limit ($Q^2 = q^2 \rightarrow \infty$, $\nu = P \cdot q \rightarrow \infty$, and $2\nu/Q^2 = z = \text{finite}$), the factorization theorem guarantees that the leading contribution to the hadron tensor, neglecting the calculable perturbative corrections, comes from the diagrams in Fig. 1 [5],

$$\begin{aligned} \hat{W}_{\mu\nu} = & 3 \sum_a e_a^2 (\hat{f}_1^a(z, Q^2) + \hat{f}_1^{\bar{a}}(z, Q^2))/z^2 \\ & \times \left[z(-g^{\mu\nu} + q^\mu q^\nu / q^2) - 2/\nu(P^\mu - \frac{z}{2}q^\mu)(P^\nu - \frac{z}{2}q^\nu) \right] \end{aligned} \quad (2)$$

where a sums over quark flavors and

$$\hat{f}_1(z, \mu^2) = \frac{1}{4}z \int \frac{d\lambda}{2\pi} e^{-i\lambda/z} \langle 0 | \not{n}_{\alpha\beta} \psi_\beta(0) | H(P) X \rangle \langle H(P) X | \bar{\psi}_\alpha(\lambda n) | 0 \rangle \quad (3)$$

is the quark fragmentation function represented by a quark-hadron four-point vertex in Fig. 1. In Eq. (3), μ^2 labels the renormalization-point dependence and the light-cone gauge $A \cdot n = 0$ has been used (otherwise a gauge link has to be explicitly included to ensure gauge invariance).

Except for a scale dependence, Eq. (2) is the naive parton-model result proposed by Feynman before QCD [8]. It resembles a similar prediction for the hadron structure functions in deep-inelastic scattering, which can be justified by the operator production expansion in QCD. However, validity of Eq. (2) in QCD is somewhat more remarkable, for the color charges are known to be confined at a scale of order 1 fm, at which something must happen to ensure the quark and antiquark keeping flying apart. The factorization theorem says whatever mechanism it is, it does not affect the hadron content of a jet.

To understand better about this independent parton fragmentation picture, we recall the way that the gluon exchanges between the quark and antiquark jets are treated when the factorization theorem is proved [9]. There are two types of gluon exchanges which are important in the so-called leading diagrams. The first is the collinear gluons emitted by a quark, with their momenta parallel to the other quark. These gluons are longitudinally polarized, and are summed to a gauge link to make Eq. (3) gauge invariant. The second is the soft gluons which either are emitted by the jets or link the two at large separations. With use of the soft-gluon approximation and the Slavnov-Taylor identities they can be factorized, and are subsequently cancelled by unitarity when final states, excluding the observed hadron, are summed.

Thus, it appears that the study of inclusive hadron production in e^+e^- annihilation reduces to evaluating Eq. (3). Notice the close similarity of $\hat{f}_1(z)$ with the quark distribution function $f_1(x)$ in a hadron of momentum P ,

$$f_1(x, \mu^2) = \frac{1}{2} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P | \bar{\psi}(0) \not{n} \psi(\lambda n) | P \rangle \quad (4)$$

Our experiences in calculating the latter provide us with valuable insights in calculating the former: First, the fact that the spectators X in Eq. (3) are colored states is not a problem in a real calculation. Similar colored intermediate states occur in the distribution functions if a complete set of states is inserted in-between the quark fields. [In the MIT bag model, these are di-quark states.] Second, the fragmentation functions at μ^2 less than 1 GeV² involve only low-energy scales and are entirely dominated by non-perturbative QCD physics. As such, techniques useful for calculating the parton distributions can in principle be used to calculate the fragmentation functions.

The explicit sum over the spectators can not be eliminated in the fragmentation functions, even if one is only interested in their moments. This renders lattice QCD and QCD sum rule methods largely useless. However, the low-energy chiral theory is an exception. One version of the theory particularly useful here is the chiral quark model of Manohar and Georgi [7], which is an effective theory of QCD at scales between $\Lambda_\chi = 4\pi f_\pi$, the chiral symmetry breaking scale, and Λ_{QCD} , the QCD confinement scale. Emergence of such a theory at low energy can be argued as follows: As an energy scale decreases below Λ_χ , the instability of the perturbative QCD vacuum leads to spontaneous breaking of the flavor $SU(3)_L \times SU(3)_R$ chiral symmetry, creating an octet of Goldstone bosons. Meanwhile, the quarks and gluons acquire their constituent masses through non-zero vacuum condensates. The interactions between the constituent quarks and gluons and Goldstone bosons are determined by chiral dynamics and are controlled by expansion of small parameters m_π^2/Λ_χ^2 and k^2/Λ_χ^2 , where k is a small momentum.

Matching the QCD quarks above $\mu = \Lambda_\chi$ and the constituent quarks below deserves some explanations. First of all, to find the exact matching conditions one has to solve both QCD and the effective theory around Λ_χ completely.

Second, an effective theory is effective only if matching conditions are simple. In this study, we take the most naive assumption that a QCD quark *is* just a constituent quark at the matching scale. This is motivated by successes of similar assumptions used in other constituent quark models. We also note that the matching conditions should be used in conjunction with the way that the effective theory is treated. We will return to this point later when we choose a cut-off for ultra-violet momenta.

For simplicity, we will neglect the gluon fragmentation at low energy scale, because in the effective theory gluons interact weakly with quarks, $\alpha_s^{\text{eff}} \sim 0.3$. This, of course, means that our result is unreliable for small z , where hadrons are mostly produced by bremsstrahlung gluons. In particular, the so-called hump-back plateau in hadron spectra, caused by intrajet coherence effects, is beyond our scope [10]. Thus to the leading order, the effective lagrangian for quarks and Goldstone bosons is

$$\mathcal{L} = \bar{\psi}(i\not{D} + \not{V} - m)\psi + g_a \bar{\psi} \not{A} \gamma_5 \psi \quad (5)$$

where ψ carries implicit color, favor, and spin indices. The vector and axial-vector fields are defined as,

$$(V_\mu, A_\mu) = \frac{i}{2}(\xi^\dagger \partial_\mu \xi \pm \xi \partial_\mu \xi^\dagger) \quad (6)$$

where $\xi = \exp(i\pi/f_\pi)$ and $\pi = \sum_a \pi^a T^a$ with $f_\pi = 93$ MeV and $\text{Tr} T^a T^b = \delta^{ab}/2$. Under the chiral transformation:

$$\begin{aligned} \Sigma (= \xi^2) &\rightarrow L \Sigma R^\dagger, \\ \xi &\rightarrow L \xi U^\dagger = U \xi R^\dagger, \\ \psi &\rightarrow U \psi, \end{aligned} \quad (7)$$

where L and R are group elements of $SU(3)_L \times SU(3)_R$, \mathcal{L} is invariant.

Let us first consider the π^+ production from a u -quark jet. The momentum-space Feynman rules for $f_1(z)$ can be derived easily when re-writing \hat{f}_1 as,

$$\hat{f}_1(z, \mu^2) = \frac{1}{4} z \int \frac{dk^- d^2 k_\perp}{(2\pi)^4} \int d^4 \xi e^{-i\xi \cdot k} \sum_X \langle 0 | \gamma_{\alpha\beta}^+ \psi_\beta(0) | H(P) X \rangle \langle H(P) X | \bar{\psi}_\alpha(\xi) | 0 \rangle \quad (8)$$

with $zk^+ = p^+$, and each matrix element is transformed to the interaction picture [11]. The lowest order diagram is shown in Fig. 2. A simple calculation yields,

$$\hat{f}_1^{\pi^+}(z) = \frac{1}{2z} g_a^2 \int \frac{dk_\perp^2}{(4\pi f_\pi)^2} \quad (9)$$

In contrast to logarithmic theories, e.g., QED and QCD, the pion transverse momentum integration has no collinear singularity. In large momentum region, it diverges quadratically. In our calculation, we cut off this type of integrations at the scale Λ_χ , beyond which the effective theory ceases to be valid. Of course, the result depends sensitively on ways that the cut is imposed, more so than in logarithmic theories. However, we believe that the arbitrariness is cancelled when a choice is used in conjunction with the corresponding matching conditions. Here, we make a simplest choice, $k_\perp^{\text{max}} = \Lambda_\chi$. Thus,

$$f_1^{\pi^+}(z, \Lambda_\chi^2) = \frac{1}{2z} g_a^2 \quad (10)$$

where $\Lambda_\chi = 4\pi f_\pi$ has been used. To confront this with experimental data, we must evolve this to appropriate scales using the Altarelli-Parisi equation [12]. In Fig. 3, we show a comparison between the evolved result ($g_a = 0.75$) and the data from EMC measurement [13]. Considering the simplicity of the approach, we think the agreement is impressive.

A more intricate case is π^- production from the u -quark jet, which is an unfavored process. In the lowest order, π^- has to be produced together with a π^+ meson. There are two way to accomplish this: The first is a sequential emission of pions through the axial-vector coupling, and the second is a sea-gull emission through the vector coupling. The two processes interfere as shown in Fig. 4. The sign of the interference term is completely determined by the sign of the vector coupling, which in turn is fixed by chiral symmetry.

The resulting expression for $\hat{f}_1(z)$ is complicated and we evaluate it numerically. A few salient features can be said briefly. First, there is a $1/z$ divergence for the longitudinal momentum integration of π^+ . A natural cut-off for

this is $m_\pi/\Lambda_\chi \sim 0.1$, the mass of pion over the scale of the virtuality of the quark. This is because pions cannot be produced with a z smaller than this due to energy conservation. Second, the numerical result shows a strong cancellation between diagrams in Fig. 4a and 4b and the interference diagrams in Fig. 4c and 4d. The cancellation is maximum if $g_a = 1/\sqrt{2}$, i.e., the vector coupling is the square of the axial-vector coupling. In Fig. 5, we have shown the EMC data and our result for $g_a = 1.0$. The fact that a slightly larger g_a is needed to reproduce the experimental data reflects the oversimplified matching conditions we use.

Finally, we present a result for the chiral-odd fragmentation function $\hat{e}(z)$, for which there is no data available. In Ref. [14], Jaffe and Ji pointed out the importance of this fragmentation function in measuring the transversity distribution of the nucleon in deep-inelastic scattering. The QCD definition for $\hat{e}(z)$ is ,

$$\hat{e}_1(z, \mu^2) = \frac{1}{4M} z \int \frac{d\lambda}{2\pi} e^{-i\lambda/z} \langle 0 | \psi_\alpha(0) | H(P) X \rangle \langle H(P) X | \bar{\psi}_\alpha(\xi) | 0 \rangle \quad (11)$$

where M is taken to be the nucleon mass. A simple calculation in the chiral quark model yields,

$$\hat{e}(z) = z \hat{f}_1(z) \frac{m_q}{M} \sim \frac{1}{3} z \hat{f}_1(z) \quad (12)$$

where m_q is the constituent quark mass. Note that this relation is only true at the scale Λ_χ , beyond which $\hat{e}(z)$ evolves in a much complicated way (twist-three) [15]. However, as a rough estimate for $\hat{e}(z)$, one can take (12) to be true beyond the model, using it in conjunction with the experimental data for $\hat{f}_1(z)$.

To summaries, we argue that the fragmentation functions can be calculated in low-energy effective theories. As an example, we show how the pion fragmentation functions are calculated in the chiral quark model. The results seem to be encouraging. A study for other fragmentation functions, including K^\pm and extending possibly to $P(\bar{P})$ productions, will be presented elsewhere.

We thank J. Collion, M. Strikman for discussions on factorization theorems and A. Manohar for discussions on the chiral quark model. Z. Zhu would like to thank Professor Da Hsuan Feng for his numerous encouragement and support.

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FIG. 1. The leading diagrams for inclusive hadron production in e^+e^- annihilation.

FIG. 2. The lowest-order diagram for π^+ production in a u -quark jet.

FIG. 3. Fragmentaion function for π^+ production in a u -quark jet. The data are taken from [13].

FIG. 4. Same as Fig. 2, for π^- production.

FIG. 5. Same as Fig. 3, for π^- production.

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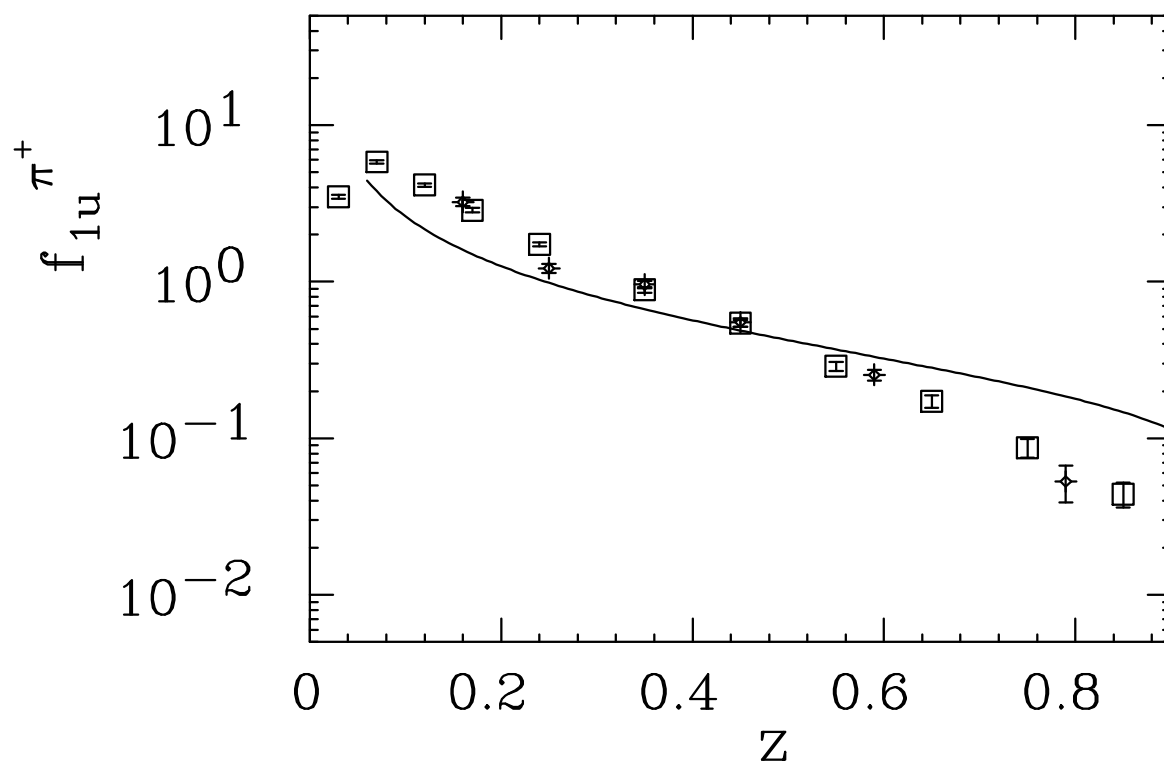


Figure 3

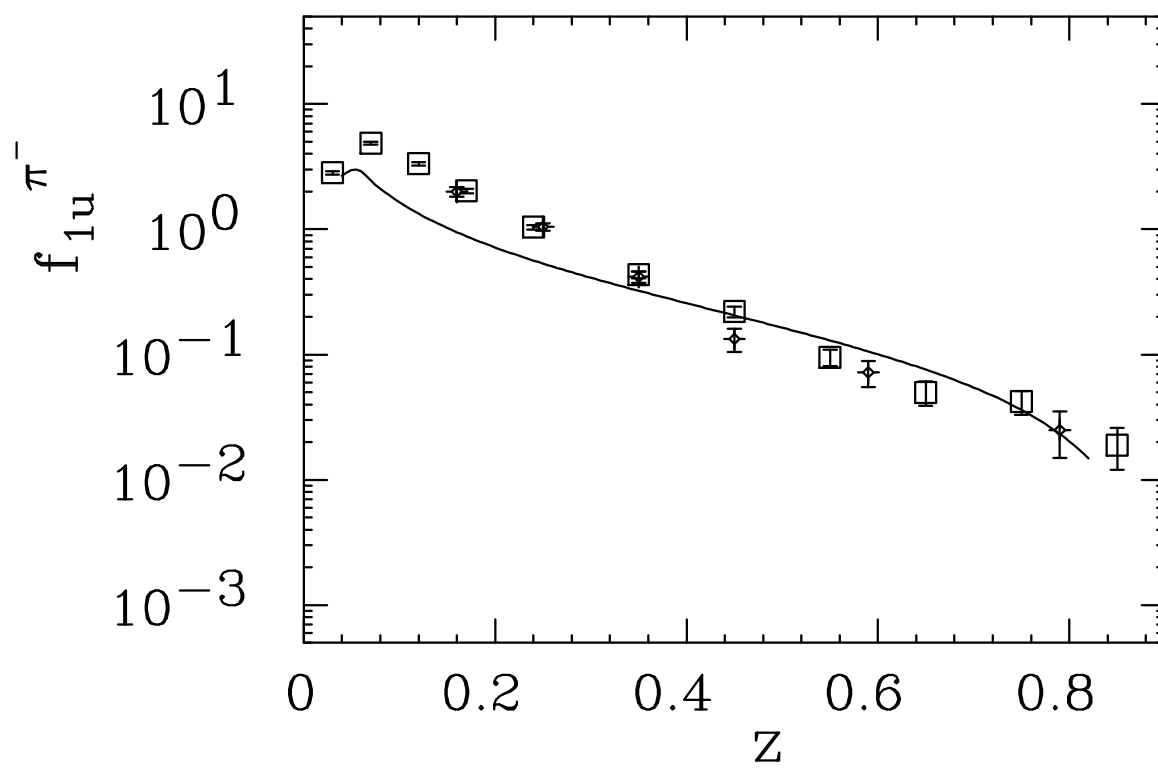


Figure 5

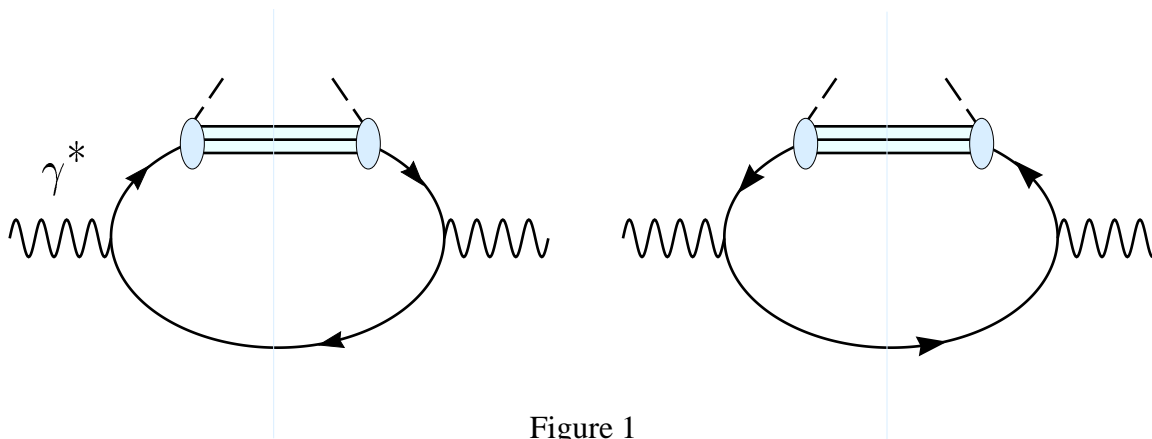


Figure 1

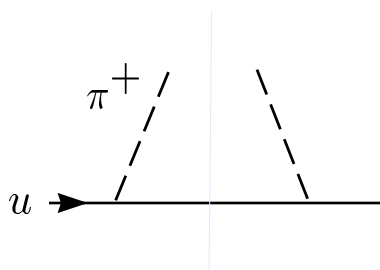


Figure 2

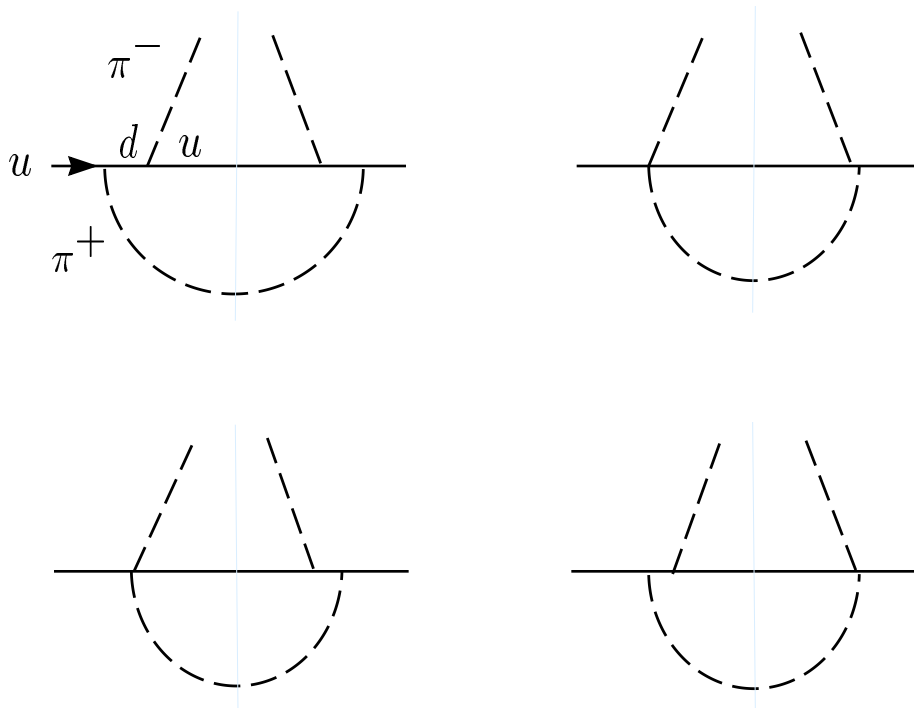


Figure 4